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## SENSITIVITY-SPECIFICITY, BAYES' RULE, AND PREDICTIVITIES

(Adapted from Basic Methods of Medical Research, Fourth Edition by Abhaya Indrayan at [amazon.in/Medical-Biostatistics-Chapman-Hall-CRC-ebook/dp/B077S4XKDW](https://www.amazon.in/Medical-Biostatistics-Chapman-Hall-CRC-ebook/dp/B077S4XKDW))

For an improved version of this topic, see Fourth Edition (2018) of the book Medical Biostatistics (CRC Press, New York), which has a large number of new topics and expanded discussion. This book available at <https://www.routledge.com/Medical-Biostatistics/Indrayan-Malhotra/p/book/9781498799539> (list price US\$129.95) or go [amazon.in/Medical-Biostatistics-Chapman-Hall-CRC-ebook/dp/B077S4XKDW](https://www.amazon.in/Medical-Biostatistics-Chapman-Hall-CRC-ebook/dp/B077S4XKDW) for discounted price

### Box 1: Simple formulas for sensitivity, specificity, positive predictivity and negative predictivity

Consider the following 2 × 2 table that gives the number of subjects with positive and negative tests, and with and without disease in a *cross-sectional random sample* from a target population. **It is only for such a sample that all four indices can be calculated from the same data.**

Test	Disease		Total
	Present	Absent	
Positive	$a$	$b$	$a+b$
Negative	$c$	$d$	$c+d$
Total	$a+c$	$b+d$	$n$

In this table, positives are in the first row and first column, and the disease confirmation is in two columns (and not rows). For using the formulas given below, ensure that the table is structured in this manner. Also note in this table that

True positives	= $a$	Persons who have disease and are test positive
True negatives	= $d$	Persons who do not have disease and are test negative
False positives	= $b$	Persons without disease but with

False negatives	= $c$	positive test Person with disease but with negative test
These give rise to the following:		
Sensitivity of the test	= $a/(a+c)$	Ability of the test to be correctly positive among those who are known to have the disease
Specificity of the test	= $d/(b+d)$	Ability of the test to be correctly negative among those who are known to be without disease.
Inherent validity of the test	= $(a+d)/n$	Ability of the test to be correctly positive or correctly negative
Positive predictivity of the test	= $a/(a+b)$	Ability of the test to correctly predict the presence of disease
Negative predictivity of the test	= $d/(c+d)$	Ability of the test to correctly predict the absence of disease
Predictive validity of the test	= $(a+d)/n$	Combined ability to correctly predict presence or absence of disease

**Example 1: Calculation of sensitivity-specificity and the two predictivities from cross-sectional data**

Joyee et al. (2003) isolated Chlamydia trachomatis by culture and also detected by direct fluorescent antibody (DFA) testing in 143 STD cases. The results obtained are as follows.

Culture	DFA		Total
	+	-	
Positive	26	1	27
Negative	9	107	116
Total	35	108	143

Although culture negativity is not a good indication of the absence of the organism but it is still considered a gold standard because of nonavailability of a better criterion. Following this convention in this example,

$$\begin{aligned} \text{sensitivity of DFA} &= 26/27 = 96.3\%, \\ \text{specificity of DFA} &= 107/116 = 92.2\%, \\ \text{positive predictivity of DFA} &= 26/35 = 74.3\%, \\ \text{negative predictivity of DFA} &= 107/108 = 99.1\%. \end{aligned}$$

The two tests have high agreement:  $(26+107)/143 = 0.93$ ; but this is mostly for negative results. DFA can replace culture to exclude the organism since the negative predictivity is so high. But the situation for detecting the presence of organism is not so satisfactory. Nine cases are culture negative but DFA positive. Positive predictivity of DFA is low.

**BAYES' RULE**

Sensitivity and specificity are easy to evaluate by a case-control study but predictivity requires that the subjects be followed up till such time that their disease status is confirmed as present or absent. This could be very time consuming and expensive. Thus predictivities are difficult to evaluate. Luckily there is a statistical procedure, called Bayes'

rule, which helps to get one from the other, provided some ancillary information is available. This is explained in Box 2 and illustrated in Example 2.

If prevalence of the disease in the target group is known, predictivity can be obtained by using sensitivity and specificity. Herein lays the importance of these two concepts. Based on confirmed cases, sensitivity and specificity are easy to obtain, and these then help to calculate diagnostically important predictivities with the help of Bayes' rule. This obviates the need to carry out expensive follow-up studies. Sensitivity and specificity of a test remain same across populations but predictivity changes depending upon the prevalence of the disease in the target population.

Dependence of predictivity on prevalence arises from putting the information in its proper context. A patient with high temperature and shivering might be diagnosed as influenza in Europe but malaria in West Africa (Chatfield 2002).

### Box 2: The significance of Bayes' rule

High fever, rigors, splenomegaly, and presence of parasite in the blood are the stages that progressively confirm malaria. As the information increases, the diagnosis becomes focused, and the probability of absence or presence of the disease firms up. The probability depends on what information is already available. The chance part is the uncovered information. The probability of any event without availability of any information is called **prior probability** and the probability after some information is available is called **posterior probability**. The latter obviously depends on the kind of information available to alter the probability.

The function of Bayes' Rule is to convert one posterior probability to its directional inverse. It converts probability of A given B to probability of B given A. This is useful to convert sensitivity of a test to its predictivity. The former is  $P(T+|D+)$  and the latter is  $P(D+|T+)$ . Such conversion is a great convenience when one probability is already available or can be easily obtained, and its inverse is otherwise difficult to obtain directly.

### Example 2: Predictivity from sensitivity/specificity and Bayes' rule

Consider hypothetical data of pap smear of 4000 apparently healthy women of age 40 years and above for cervical cancer and each independently evaluated for histology also through biopsy as a gold test. The following results are obtained in this cross-sectional study.

Pap smear	Cervical cancer by histology		Total
	Present	Absent	
Positive	190	210	400
Negative	10	3590	3600
Total	200	3800	4000

The results of the gold test (histology) are in two columns of this table whereas they are in two rows in the table in Example 1. In this example, pap smear is false positive in 210 cases and false negative in 10 cases. Using the definitions given earlier,

sensitivity of pap smear	= $190/200 = 95.0\%$ ;
specificity of pap smear	= $3590/3800 = 94.5\%$ ;
positive predictivity of pap smear	= $190/400 = 47.5\%$ ;
negative predictivity of pap smear	= $3590/3600 = 99.7\%$ .

Despite extremely high sensitivity and specificity (both nearly 95 percent), the positive predictivity of pap smear is still low (less than 50 percent). Thus this is not a good diagnostic tool for cervical cancer in this population although it is extremely good in ruling out the disease (negative predictivity 99.7 percent). This is what makes it a good screening test. Thus good sensitivity and specificity do not necessarily make a good diagnostic test.

The prevalence of cervical cancer in this group of subjects is  $P(D+) = 200/4000 = 0.05$ . Bayes' rule says that

$$\begin{aligned} \text{Positive predictivity} &= \frac{\text{Sensitivity} \times \text{Prevalence}}{\text{Sensitivity} \times \text{Prevalence} + (1 - \text{Specificity}) \times (1 - \text{Prevalence})} \\ &= \frac{0.95 \times 0.05}{0.95 \times 0.05 + (1 - 0.945) \times (1 - 0.05)} \\ &= 0.476 \text{ or } 47.6\%. \end{aligned}$$

This is nearly the same as obtained earlier directly from the data also. Minor difference is due to decimal approximation in the calculation of specificity, which more exactly is 94.4737 percent but we used 94.5 percent. Similar equivalence can be shown for negative predictivity also.

### WHERE TO USE WHICH TEST?

As already mentioned, a test with good positive predictivity is required for confirming the presence of disease, and a test with good negative predictivity to exclude the disease. Screening tests, which are used at initial stages, or in a community setup, should be able to correctly exclude all negative cases. Cervical cytology (Pap smear) is used for screening of cervical cancer, and mammography for breast cancer. Blood glucose level is used for screening of diabetes, and Mantoux test for tuberculosis infection. They are tests with high negative predictivity. When the test is negative it is safe to assume in these cases that the disease is absent. In addition, a screening test must be durable to withstand constant use and considerable abuse because it tends to be used on a large number of subjects.

The goal of diagnosis is to ferret out cases while keeping false positives to a negligible level. False positivity can create an unmanageable backlog of cases that actually does not require medical attention. This can lead to organizational fatigue and the staff may wear out, causing loss of alertness. Thus a diagnostic test is good if it has high positive predictivity. Histology is good to confirm malignancy so is x-ray for bone fracture. Try to use such tests in your work.

### REFERENCES

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